Solutions to Sample Midterm III

(1) (i) $\int 3xe^{4x^2}\,dx$

$u = 4x^2$

$du = 8x\,dx$

$\Rightarrow \frac{1}{8}du = x\,dx$

$= \frac{3}{8}\int e^u\,du$

$= \frac{3}{8}e^u + C = \boxed{\frac{3}{8}e^{4x^2} + C}$

(ii) $\int x\cos(x^2+2)\,dx$

$u = x^2+2$

$du = 2x\,dx$

$\Rightarrow \frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int \cos(u)\,du$

$= \frac{1}{2}\sin(u) + C = \boxed{\frac{1}{2}\sin(x^2+2) + C}$

(2) $\displaystyle\int_e^{e^2} \frac{dx}{x\ln x}$ $\qquad u = \ln x$

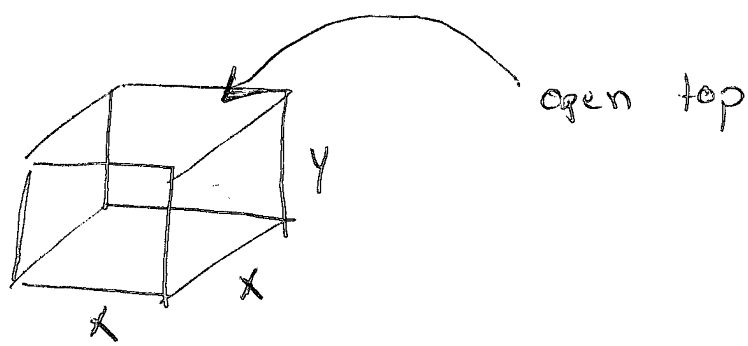
$\qquad\qquad\qquad\qquad du = \frac{1}{x}\,dx$

$$= \int_1^2 \frac{1}{u}\,du$$

(Notice change in upper/lower limits)

$$= \ln|u|\Big]_{u=1}^{2} = \ln 2 - \ln 1 = \boxed{\ln 2}$$

(3)



Surface area $= 1200 = x^2 + 4xy$

$$\Rightarrow y = \frac{1200 - x^2}{4x}$$

Volume $= V = x^2 y = x^2\left(\frac{1200 - x^2}{4x}\right)$

$$= \frac{x}{4}\left(1200 - x^2\right)$$

$$= 300x - \frac{x^3}{4}$$

$$\Rightarrow \frac{dV}{dx} = 300 - \frac{3}{4}x^2$$

$$0 = \frac{dV}{dx} = 300 - \frac{3}{4}x^2 \iff \frac{3}{4}x^2 = 300$$

$x = \pm 20$ critical points

Take positive root

Sign of $\frac{dV}{dx}$ $\quad \underset{20}{+ + \mid - -}$, so volume

maximized at $x = 20$, $y = \frac{1200 - 400}{80} = 10$

$\Rightarrow$ largest possible volume $= 20^2 \cdot 10 = \boxed{4000 \text{ cm}^3}$

(4) $f''(x) = 1 - 6x + 48x^2$, $f(0) = 1$, $f'(0) = 2$

$$f'(x) = \int f''(x)\,dx = x - 3x^2 + 16x^3 + C$$

$$2 = f'(0) = C$$

$$\Rightarrow f'(x) = x - 3x^2 + 16x^3 + 2$$

$$f(x) = \int f'(x)\,dx = \frac{1}{2}x^2 - x^3 + 4x^4 + 2x + D$$

$1 = f(0) = D$

$$\Rightarrow \boxed{f(x) = \frac{1}{2}x^2 - x^3 + 4x^4 + 2x + 1}$$

(5) (a) $v(t) = t^2 - 5t + 6$

$$\text{Displacement} = \int_0^3 v(t)\,dt = \int_0^3 (t^2 - 5t + 6)\,dt$$

$$= \frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t \Big]_{t=0}^{3}$$

$$= \frac{27}{3} - \frac{5 \cdot 9}{2} + 18$$

$$= \boxed{\frac{9}{2}}$$

(b) $$\text{Distance} = \int_0^3 |v(t)|\,dt$$

$$= \int_0^2 (t^2 - 5t + 6)\,dt$$

$$+ \int_2^3 -(t^2 - 5t + 6)\,dt$$

$0 = v(t) = t^2 - 5t + 6$

$0 = (t-2)(t-3)$

zeros @ $t = 2, 3$

sign of $v(t)$: $+\ +$ on $(0, 2)$, $-\ -$ on $(2, 3)$

0 2 3

$$= \left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t\right]_{t=0}^{2} - \left[\frac{1}{3}t^3 - \frac{5}{2}t^2 + 6t\right]_{t=2}^{3}$$

$$= 2\left[\frac{8}{3} - \frac{20}{2} + 12\right] - \left[\frac{27}{3} - \frac{5\cdot 9}{2} + 18\right]$$

$$= \boxed{\frac{29}{6}}$$

(6) (a) $\frac{d}{dt}(t\ln t - t) = \ln t + 1 - 1 = \ln t$

$\Rightarrow$ $t\ln t - t$ is antiderivative for $\ln t$

(b) $\int_1^{\sqrt{e}} x\ln x^2\,dx$

$u = x^2$

$du = 2x\,dx$

$\Rightarrow \frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\int_1^e \ln u\,du$$

$$= \frac{1}{2}\left[u\ln u - u\right]_{u=1}^{u=e} = \frac{1}{2}(e - e) - \frac{1}{2}(0 - 1)$$

$$= \boxed{\frac{1}{2}}$$